

ラグランジュの運動方程式 ( 2 )

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = T$$

$$L = K_1 + K_2 - P_1 - P_2$$

K 運動エネルギー

P ポテンシャルエネルギー

アーム 1

L1 : アームの長さ

L1c : ピボット A から重心までの  
距離

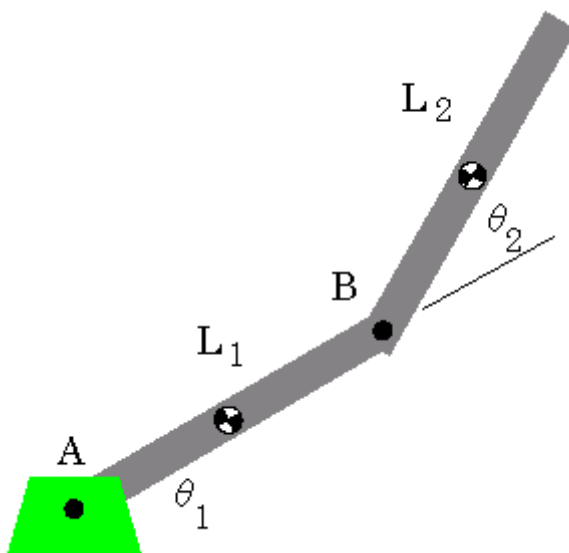
m1 : 質量

J1c : 重心まわりの慣性 2 次モー  
メント

アーム 2

L2c : ピボット B から重心までの  
距離

(他はアーム 1 参照)



ピボットでのトルクを求める運動方程式をたてよう。

アーム 1 にたいして、

$$K_1 = \frac{1}{2} m_1 \cdot (L_{c1} \cdot \dot{\theta}_1)^2 + \frac{1}{2} J_{c1} \cdot \dot{\theta}_1^2$$

$$P_1 = m_1 \cdot g \cdot L_{c1} \cdot S_1$$

アーム 2 にたいして、

$$x_{c2} = L_1 \cdot C_1 + L_{c2} \cdot \cos(\theta_1 + \theta_2)$$

$$v_{c2x} = -L_1 \cdot S_1 \cdot \omega_1 - L_{c2} \cdot \sin(\theta_1 + \theta_2) \cdot (\omega_1 + \omega_2)$$

$$v_{c2x}^2 = L_1^2 \cdot S_1^2 \cdot \omega_1^2 + L_{c2}^2 \cdot (\sin(\theta_1 + \theta_2))^2 \cdot (\omega_1 + \omega_2)^2 \dots \\ + 2 \cdot L_1 \cdot L_{c2} \cdot S_1 \cdot \sin(\theta_1 + \theta_2) \cdot (\omega_1 + \omega_2) \cdot \omega_1 \quad (a)$$

$$y_{c2} = L_1 \cdot S_1 + L_{c2} \cdot \sin(\theta_1 + \theta_2)$$

$$v_{c2y} = L_1 \cdot C_1 \cdot \omega_1 + L_{c2} \cdot \cos(\theta_1 + \theta_2) \cdot (\omega_1 + \omega_2)$$

$$v_{c2y}^2 = L_1^2 \cdot C_1^2 \cdot \omega_1^2 + L_{c2}^2 \cdot (\cos(\theta_1 + \theta_2))^2 \cdot (\omega_1 + \omega_2)^2 \dots \\ + 2 \cdot L_1 \cdot L_{c2} \cdot C_1 \cdot \cos(\theta_1 + \theta_2) \cdot (\omega_1 + \omega_2) \cdot \omega_1 \quad (b)$$

ところで、

$$S_1 \cdot \sin(\theta_1 + \theta_2) = S_1 \cdot (S_1 \cdot C_2 + S_2 \cdot C_1) = S_1^2 \cdot C_2 + S_1 \cdot S_2 \cdot C_1$$

$$C_1 \cdot \cos(\theta_1 + \theta_2) = C_1 \cdot (C_1 \cdot C_2 - S_1 \cdot S_2) = C_1^2 \cdot C_2 - S_1 \cdot S_2 \cdot C_1$$

であるから、(a)、(b)から

$$v_{c2}^2 = v_{c2x}^2 + v_{c2y}^2 \\ = L_1^2 \cdot \omega_1^2 + L_{c2}^2 \cdot (\omega_1 + \omega_2)^2 + 2 \cdot L_1 \cdot L_{c2} \cdot (C_2) \cdot (\omega_1^2 + \omega_1 \cdot \omega_2)$$

アーム 2 にたいして、

$$K_2 = \frac{1}{2} m_2 \cdot v_{c2}^2 + \frac{1}{2} J_{c2} \cdot (\omega_1 + \omega_2)^2$$

$$P_2 = m_2 \cdot g \cdot (L_1 \cdot S_1 + L_{c2} \cdot S_{12})$$

ラグランジュの式を小分けしてもとめよう。

$$\frac{\partial K_1}{\partial \omega_1} = m_1 \cdot L_{c1}^2 \cdot \omega_1 + J_{c1} \cdot \omega_1$$

$$\frac{d}{dt} \left( \frac{\partial K_1}{\partial \omega_1} \right) = (m_1 L_{c1}^2 + J_{c1}) \dot{\omega}_1$$

$$\frac{\partial K_1}{\partial \theta_1} = 0$$

$$\frac{\partial P_1}{\partial \omega_1} = 0$$

$$\frac{\partial P_1}{\partial \theta_1} = m_1 g L_{c1} C_1$$

$$\frac{\partial K_2}{\partial \omega_1} = m_2 \left( L_1^2 \omega_1 + L_{c2}^2 (\omega_1 + \omega_2) + L_1 L_{c2} C_2 (2\omega_1 + \omega_2) \right) + J_{c2} (\omega_1 + \omega_2)$$

$$\frac{d}{dt} \left( \frac{\partial K_2}{\partial \omega_1} \right) =$$

$$= m_2 \left( L_1^2 \dot{\omega}_1 + L_{c2}^2 (\dot{\omega}_1 + \dot{\omega}_2) + L_1 L_{c2} C_2 (2\dot{\omega}_1 + \dot{\omega}_2) - L_1 L_{c2} S_2 \omega_2 (2\omega_1 + \omega_2) \right) + J_{c2} (\dot{\omega}_1 + \dot{\omega}_2)$$

$$= \left( m_2 L_1^2 + m_2 L_{c2}^2 + 2m_2 L_1 L_{c2} C_2 + J_{c2} \right) \dot{\omega}_1 + \left( m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2} \right) \dot{\omega}_2$$

$$- m_2 L_1 L_{c2} S_2 \omega_2 (2\omega_1 + \omega_2)$$

$$\frac{\partial K_2}{\partial \theta_1} = 0$$

$$\frac{\partial P_2}{\partial \omega_1} = 0$$

$$\frac{\partial P_2}{\partial \theta_1} = m_2 g (L_1 C_1 + L_{c2} C_{12})$$

ゆえに

$$\begin{aligned}
T_1 &= \frac{d}{dt} \left( \frac{\partial}{\partial \omega_1} (K_1 + K_2 - P_1 - P_2) \right) - \frac{\partial}{\partial \theta_1} (K_1 + K_2 - P_1 - P_2) \\
&= \left( m_1 L_{c1}^2 + J_{c1} + m_2 L_1^2 + m_2 L_{c2}^2 + 2m_2 L_1 L_{c2} C_2 + J_{c2} \right) \dot{\omega}_1 \\
&\quad + \left( m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2} \right) \dot{\omega}_2 - m_2 L_1 L_{c2} S_2 \omega_2 (2\omega_1 + \omega_2) \\
&\quad + m_1 g L_{c1} C_1 + m_2 g (L_1 C_1 + L_{c2} C_{12})
\end{aligned}$$

つぎに

$$\frac{\partial K_1}{\partial \omega_2} = 0$$

$$\frac{d}{dt} \left( \frac{\partial K_1}{\partial \omega_2} \right) = 0$$

$$\frac{\partial P_1}{\partial \omega_2} = 0$$

$$\frac{\partial P_1}{\partial \theta_2} = 0$$

$$\frac{\partial K_2}{\partial \omega_2} = m_2 \left( L_{c2}^2 (\omega_1 + \omega_2) + L_1 L_{c2} C_2 \omega_1 \right) + J_{c2} (\omega_1 + \omega_2)$$

$$\frac{d}{dt} \left( \frac{\partial K_2}{\partial \omega_2} \right) = m_2 \left( L_{c2}^2 (\dot{\omega}_1 + \dot{\omega}_2) + L_1 L_{c2} C_2 \dot{\omega}_1 - L_1 L_{c2} S_2 \omega_2 \omega_1 \right) + J_{c2} (\dot{\omega}_1 + \dot{\omega}_2)$$

$$= \left( m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2} \right) \dot{\omega}_1 + \left( m_2 L_{c2}^2 + J_{c2} \right) \dot{\omega}_2 - m_2 L_1 L_{c2} S_2 \omega_1 \omega_2$$

$$\frac{\partial K_2}{\partial \theta_2} = m_2 L_1 L_{c2} S_2 \left( \omega_1^2 + \omega_1 \omega_2 \right)$$

$$\frac{\partial P_2}{\partial \omega_2} = 0$$

$$\frac{\partial P_2}{\partial \theta_2} = m_2 g L_{c2} C_{12}$$

ゆえに

$$\begin{aligned} T_2 &= \frac{d}{dt} \left( \frac{\partial}{\partial \omega_2} (K_1 + K_2 - P_1 - P_2) \right) - \frac{\partial}{\partial \theta_2} (K_1 + K_2 - P_1 - P_2) \\ &= (m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2}) \dot{\omega}_1 + (m_2 L_{c2}^2 + J_{c2}) \dot{\omega}_2 + m_2 L_1 L_{c2} S_2 \omega_1^2 + m_2 g L_{c2} C_{12} \end{aligned}$$

以上をまとめて、

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}$$

ただし、

$$M_{11} = (m_1 L_{c1}^2 + J_{c1} + m_2 L_1^2 + m_2 L_{c2}^2 + 2m_2 L_1 L_{c2} C_2 + J_{c2})$$

$$M_{12} = (m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2})$$

$$M_{21} = (m_2 L_{c2}^2 + m_2 L_1 L_{c2} C_2 + J_{c2})$$

$$M_{22} = (m_2 L_{c2}^2 + J_{c2}) \dot{\omega}_2$$

$$c_1 = -m_2 L_1 L_{c2} S_2 \omega_2 (2\omega_1 + \omega_2)$$

$$c_2 = m_2 L_1 L_{c2} S_2 \omega_1^2$$

$$g_1 = m_1 g L_{c1} C_1 + m_2 g (L_1 C_1 + L_{c2} C_{12})$$

$$g_2 = m_2 g L_{c2} C_{12}$$