

## 第 1 章 複素平面と $e^{i\theta} = \cos \theta + i \cdot \sin \theta$

1 . 3  $z = 2 \cdot e^{i\frac{\pi}{6}}$  のとき  $\text{Im}(z)=1$   $\text{Arg}(z) = \pi/6$   $|z|=2$

1 . 4  $z_1 = 2 \cdot e^{i\frac{\pi}{6}}$  ,  $z_2 = 3 \cdot e^{i\frac{\pi}{3}}$  のとき

$$(1) z = z_1 \cdot z_2 = 6e^{i\frac{\pi}{2}} \quad (2) z = \frac{z_2}{z_1} = \frac{3}{2}e^{i\frac{\pi}{6}}$$

1 . 5  $z = 1 + i \cdot \sqrt{3} = 2e^{i\frac{\pi}{3}}$  のとき

$$(1) u = i \cdot z = -\sqrt{3} + i = 2e^{i\frac{5}{6}\pi} \quad (2) u = -z = -1 - i\sqrt{3} = 2e^{i\frac{4}{3}\pi}$$

$$(3) u = (\sqrt{3} + i) \cdot z = 4i = 4e^{i\frac{\pi}{2}}$$

$$(4) u = \frac{z}{1+i} = \frac{1+i\sqrt{3}}{1+i} \frac{1-i}{1-i} = \frac{1+\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2}i = \sqrt{2}e^{i\frac{\pi}{12}}$$

$$\text{または } u = \frac{z}{1+i} = \frac{2e^{i\frac{\pi}{3}}}{\sqrt{2}e^{i\frac{\pi}{4}}} = \sqrt{2}e^{i\frac{\pi}{12}}$$

1 . 6  $x^5 + 1 = 0$  の根は 5 つで  $e^{i\frac{1}{5}\pi}$   $e^{i\frac{3}{5}\pi}$   $e^{i\pi}$   $e^{i\frac{7}{5}\pi}$   $e^{i\frac{9}{5}\pi}$  。

5 つの根を x に代入して  $x^5 = -1$  となることを確かめるとよい。

1 . 7 オイラーの公式を使うと

$$e^{iA}e^{iB} = e^{i(A+B)} = \cos(A+B) + i\sin(A+B)$$

また

$$e^{iA}e^{iB} = (\cos A + i \sin A)(\cos B + i \sin B) = \cos A \cos B - \sin A \sin B + i(\sin A \cos B + \cos A \sin B)$$

両者の実部と虚部を比較して

$$(1) \quad \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$(2) \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$1.8 \quad z' = ze^{i\theta} = (x+iy)(\cos \theta + i \sin \theta) = \cos \theta \cdot x - \sin \theta \cdot y + i(\sin \theta \cdot x + \cos \theta \cdot y)$$

$$\text{ゆえに} \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

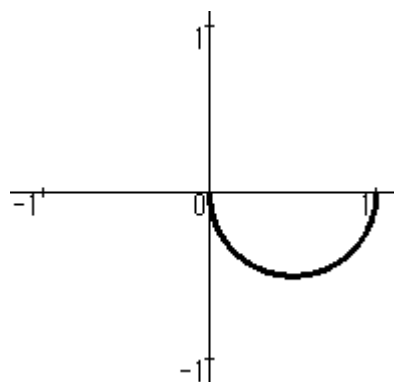
$$1.9 \quad u = ze^{-i\theta} = (x+iy)(\cos \theta - i \sin \theta) = \cos \theta \cdot x + \sin \theta \cdot y + i(-\sin \theta \cdot x + \cos \theta \cdot y)$$

$$\text{ゆえに} \quad \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

1.10 (周波数伝達関数 Frequency transfer function のベクトル軌跡)

(1)のベクトル軌跡

サンプルプログラム



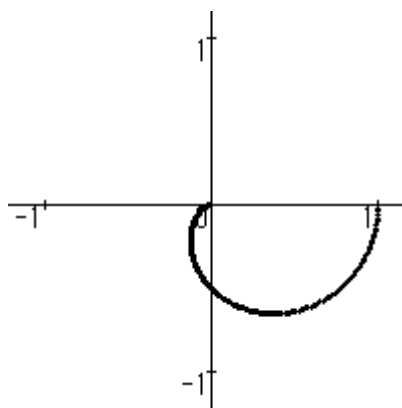
```
!振掃 02d.bas
option arithmetic complex
set window -3,3,-3,3
set axis color 1
draw axes

let j=complex(0,1)
def G(s)=1/(s+1)

set line width 3
for w=0 to 1000 step 0.01
  let s=j*w
  let H=G(s)
  plot lines:Re(H),Im(H)
next w
END
```

(2)のベクトル軌跡

サンプルプログラム



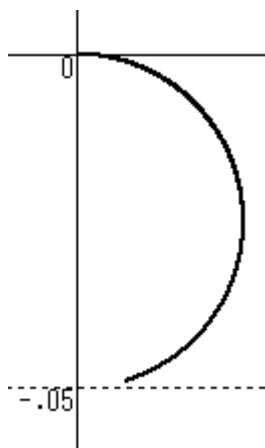
```
!振掃 02c.bas
option arithmetic complex
set window -3,3,-3,3
set axis color 1
draw axes
let j=complex(0,1)
def G(s)=1/(s^2+2*s+1)
set line width 3
for w=0 to 1000 step 0.01
  let s=j*w
  let H=G(s)
  plot lines:Re(H),Im(H)
next w
END
```

1 . 1 1  $r_1 = 1.5$  ,  $r_2 = 1.5$  ,  $x_1 = 10$  , and  $x_2 = 10$ . のとき  $s=0.0001 \sim 1$  のときの

$$G = \frac{1}{r_1 + \frac{r_2}{s} + j \cdot (x_1 + x_2)}$$

のベクトル軌跡は

ベクトル軌跡



サンプルプログラム

```
!振掃 02e.bas
option arithmetic complex
set window -3,3,-3,3
set axis color 1
draw axes
let j=complex(0,1)
let r1=1.5
let r2=1.5
let x1=10
let x2=10
def F(s)=r1+r2/s+j*(x1+x2)

set line width 2
for s=0.0001 to 1 step 0.0001
  let H=1/F(s)
  plot lines:Re(H),Im(H);
next s
END
```

1 . 1 2

A , B , C , D 点で  $w = \frac{e^z}{z+1} \rightarrow 0$